APPENDIX XII:

RELATIVE DENSITY

1. GENERAL. Relative density expresses the degree of compactness of a cohesionless soil with respect to the loosest and the densest conditions of the soil that can be attained by specified laboratory procedures; a soil in the loosest state would have a relative density of 0 percent and in the densest state, of 100 percent. The dry unit weight of a cohesionless soil does not, by itself, reveal whether the soil is loose or dense, due to the influence of particle shape and gradation on this property. Only when viewed against the possible range of variation, in terms of relative density, can the dry unit weight be related to the compaction effort used to place the soil in an embankment or indicate the volume-change tendency of the soil when subjected to shear stresses. Originally, relative density, D_d, was defined by the equation

$$D_d,\ \text{percent} = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100$$

where

e_{max} = void ratio of the soil in the loosest state which can be attained in the laboratory

e = void ratio of the soil in place

e_{min} = void ratio of the soil in the densest state which can be attained in the laboratory

though, for ease of computation, relative density may be expressed in terms of dry unit weight by the equation

$$D_d,\ \text{percent} = \frac{\gamma_d - \gamma_{d\ min}}{\gamma_{d\ max} - \gamma_{d\ min}} \times \frac{\gamma_{d\ max}}{\gamma_d} \times 100$$

where

γ_d = dry unit weight of the soil in place, called "in-place density

$\gamma_{d\ min}$ = dry unit weight of the soil in the loosest state which can be attained in the laboratory, called "minimum density "

$\gamma_{d\ max}$ = dry unit weight of the soil in the densest state which can be attained in the laboratory, called "maximum density."

The in-place density of a soil may be determined by various methods in the field or from undisturbed samples brought into the laboratory; its determination will not be considered in this test procedure which covers only the methods for determining the minimum and maximum densities. There are few difficulties in determining minimum density by a standard method, but restrictions imposed by the availability of special apparatus under certain conditions or the character of the soil may not permit the determination of maximum density by a single, standard method to the exclusion of all others. However, the method described herein, employing a vibratory table, shall be preferred to all other methods;† the alternative method for attaining the maximum denseness of a soil by hammering on the mold (as described in Appendix XIIA, MODIFIED PROVIDENCE VIBRATED DENSITY TEST) should be followed only when the use of the vibratory table is not feasible. The method used for determining the maximum density must be reported when presenting relative density data.

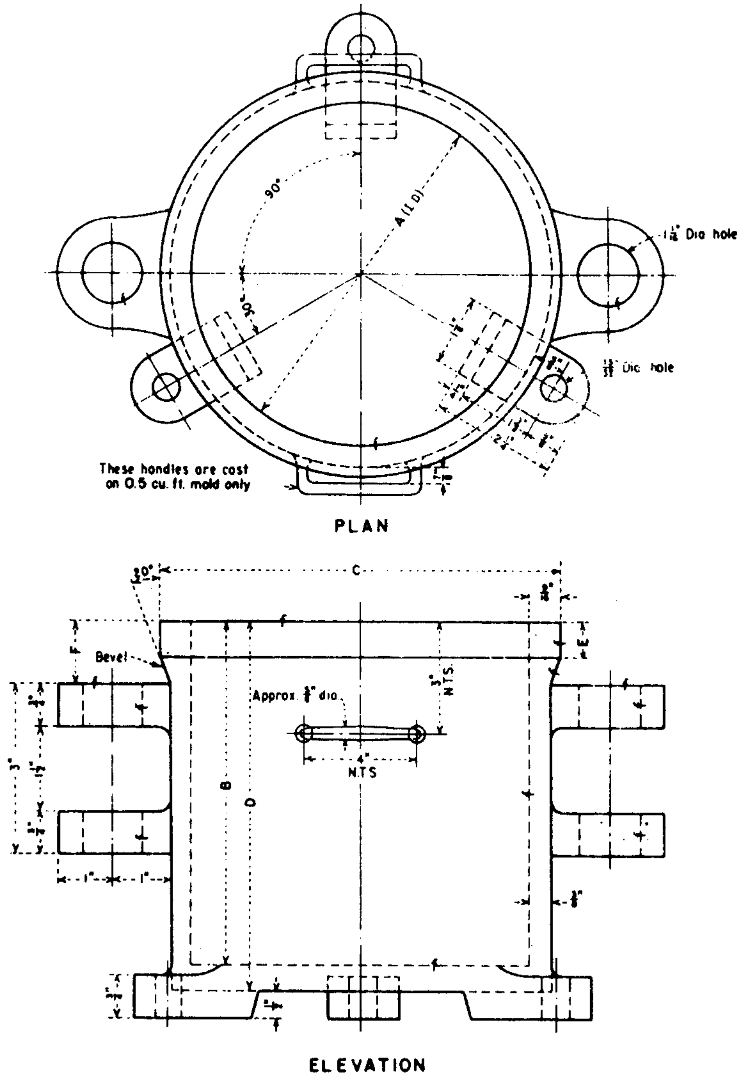
2. APPARATUS. The apparatus for determining the minimum and maximum densities of a cohesionless soil shall consist of the following:

a. Cylindrical molds or measures of 0.1-cu-ft capacity (6-in. inside diameter) and 0.5-cu-ft capacity (11-in. inside diameter), as shown in Figure 1.‡ The molds should be cast of silicon aluminum alloy ASTM-SG70A.

b. Surcharge assemblies, to fit each size mold, as shown in

† Other methods are currently being investigated to overcome the problem of segregation encountered in some soils using the vibratory table.

‡ These molds, as well as the surcharge assemblies and dial indicator holder shown in Figure 2, are available commercially from Stebbins Mfg. & Supply co., 1733 Blake St., Denver, Colo.



MOLD SIZE (CU FT)	DIMENSIONS					
	A	B	C	D	E	F
0.1	6.000"	6.112"	$7\frac{1}{8}$"	$6\frac{1}{2}$"	[illegible]	$1\frac{1}{8}$"
0.5	11.000"	9.092"	$12\frac{1}{8}$"	$9\frac{1}{4}$"	[illegible]	2"

Figure 1. Details of molds

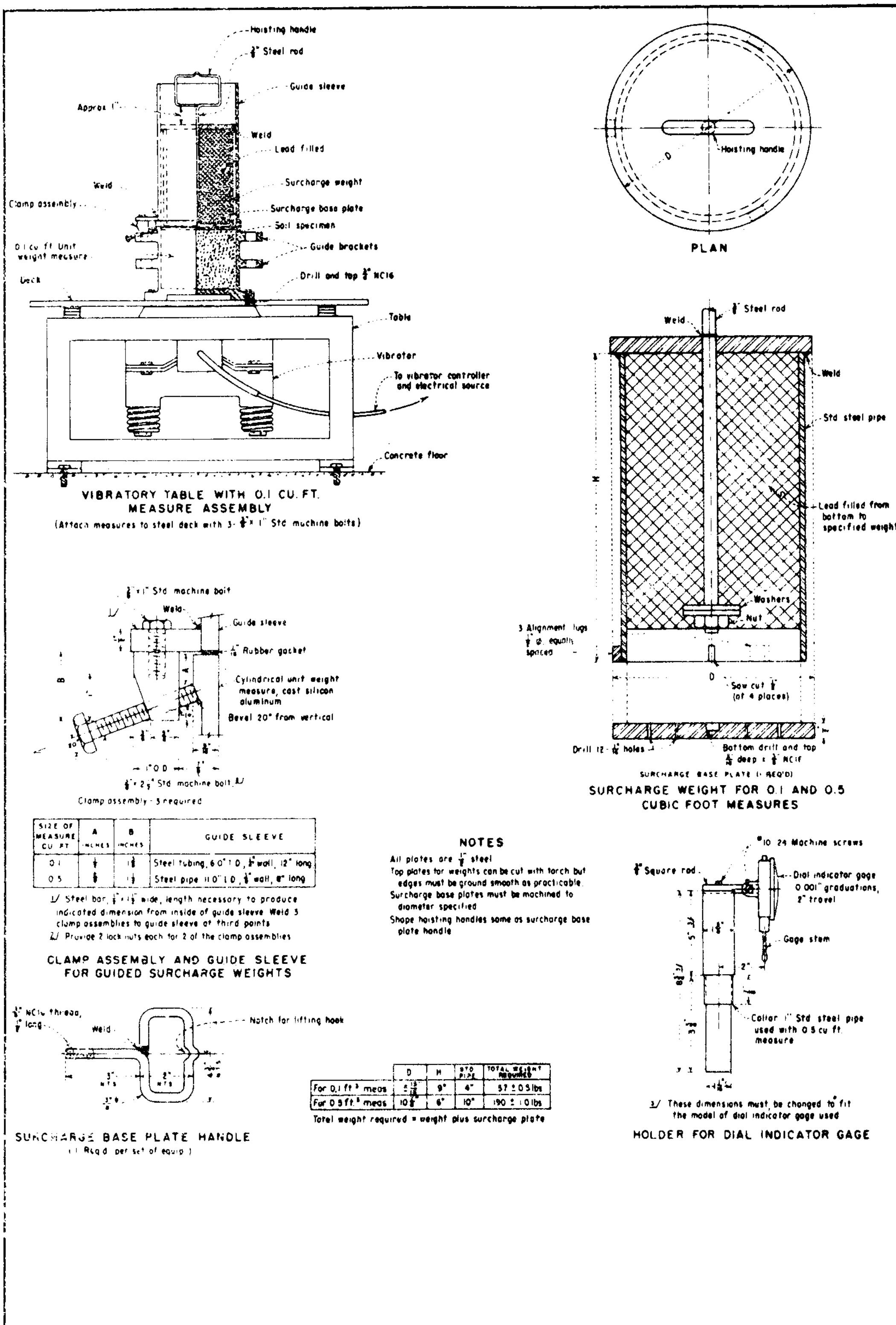


Figure 2. Equipment for maximum density determination

Figure 2. Each assembly shall include a surcharge weight (equivalent to 2 psi), a surcharge baseplate with handle and a guide sleeve with clamp assemblies.

c. Dial indicator, having 0.001-in. graduations and a 2-in. range.

d. Holder, for dial indicator, with collar, as shown in Figure 2.

e. Calibration bar, metal, 3 in. by 1/8 in. by 12 in. long.

f. Vibratory table, as shown in Figure 2, with a cushioned steel vibrating deck about 30 by 30 in., actuated by an electromagnetic vibrator and mounted to a concrete floor or slab of large mass. The vibrator should be a seminoiseless type with a net weight over 100 lb. The vibrator shall have a frequency of 3600 vibrations per minute and variable vibrator amplitudes to a maximum of at least 0.015 in. under a 250-lb load, and be suitable for use with 230-volt alternating current.†

g. Hoist, having a capacity of at least 300 lb, for handling the heavier surcharge weight.

h. Pouring device, as shown in Figure 3, which has metal funnels with 1/2- and 1-in.-diameter cylindrical spouts, each attachable to a metal can 6 in. in diameter by 12 in. high.

i. Hand scoop, large, metal.

j. Mixing pans, large, metal.

k. Sample splitter or riffle.

l. Straightedge, 15-in., steel.

m. Platform scales, having a capacity

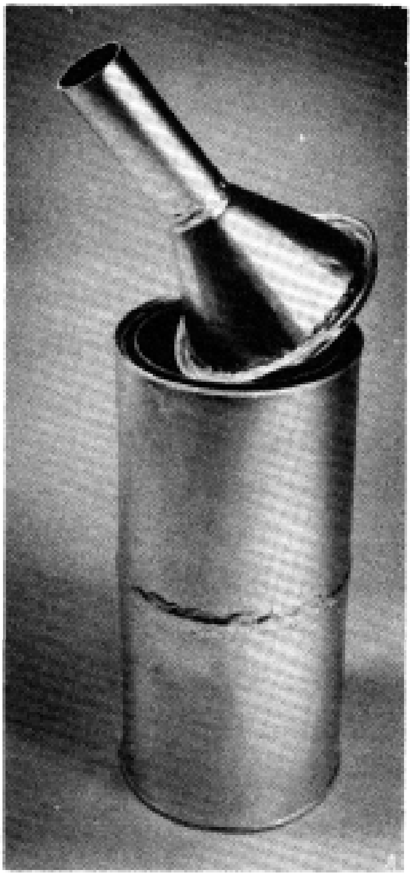


Figure 3. Pouring device, with i-in.-diameter spout

† The Syntron VP-80 and VP-240 Vibratory Packers, manufactured by Syntron Co., Homer City, Pa., have proven satisfactory. (The VP-80 has been replaced by a later model, VP-86, with the same characteristics.)

of at least 100 lb and sensitive to 0.01 lb.

n. Oven (see Appendix I, WATER CONTENT - GENERAL).

3. CALIBRATION OF EQUIPMENT. Each mold must be calibrated as follows:

a. Determine the weight, W_m, of each mold to the nearest 0.01 lb.

b. Determine the capacity, or total inside volume, V_m, of the 0.1-cu-ft mold to the nearest 0.0001 cu ft and of the 0.5-cu-ft mold to the nearest 0.001 cu ft.

c. Determine the inside cross-section area, A_m, at the open end of the 0.1-cu-ft mold to the nearest 0.0001 sq ft and of the 0.5-cu-ft mold to the nearest 0.001 sq ft.

d. Determine the initial dial reading, h_o, for each mold and surcharge baseplate combination in the following manner:

(1) Measure the thickness of the calibration bar, t_c, and of the surcharge baseplate, t_s, to the nearest 0.001 in.

(2) Lay the calibration bar across the top of the mold along the axis defined by the brackets for the dial indicator holder, as shown in Figure 4a, see page 9.

(3) Insert the dial indicator holder into the brackets on one side of the mold so that the dial indicator stem rests on the calibration bar, and note the dial reading. Then insert the dial indicator holder into the brackets on the opposite side of the mold and note the dial reading. The dial indicator holder should be placed in the same position in the guide brackets for each reading by means of matchmarks on the holder and on the brackets.

(4) Compute the average, h_r, of the two dial readings.

(5) Compute the initial dial reading, h_o, by the equation

$$h_o = h_r + t_c - t_s$$

4. PREPARATION OF SAMPLE. The soil to be tested must be oven-dried

and then permitted to cool in an airtight container. Aggregations of fine particles shall be thoroughly broken and a representative sample removed from the soil using a sample splitter or riffle. The representative sample should weigh at least 25 lb if the maximum particle size is less than 1-1/2 in. and at least 100 lb if the maximum particle size is between 1-1/2 and 3 in. If the sample contains more than 10 percent by weight particles larger than 3 in. the determination of the maximum and minimum density becomes a test of a research nature.

5. PROCEDURE. Every precaution must be observed while handling the sample to prevent segregation and to preserve the oven-dried condition. The minimum and maximum densities shall be determined using the same sample, by first placing the soil into a mold in the loosest possible state to attain the minimum density and then vibrating it into the densest possible state to attain the maximum density. The 0.1-cu-ft mold shall be used for the determinations if the maximum particle size is less than 1-1/2 in. and the 0.5-cu-ft mold shall be used if the maximum particle size is between 1-1/2 and 3 in.

a. Minimum Density Determination. The procedure for determining the minimum density shall consist of the following steps:

(1) Record all identifying information for the sample. such as project, boring number, etc., on a data sheet (Plate XII-1 is a suggested form).

(2) Record the weight, inside volume, and end area of the mold on the data sheet.

(3) Carefully place the oven-dried soil into the mold in the loosest possible condition.† Fill the mold in layers, using the pouring device for material having maximum particle sizes less than 3/8 in. or the hand scoop for material having larger particle sizes. Exercise the greatest care at all times to avoid jarring the mold or otherwise disturbing the previously placed layers. When using the pouring device (with the

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† If the maximum density of the oven-dried sample is to be determined also, the mold may be attached to the deck of the vibratory table before filling (see paragraph 5*b*(2)).

1/2-in.-diameter spout if the maximum particle size passes the No. 4 sieve and the 1-in.-diameter spout if the maximum particle size is between the No. 4 sieve and 3/8-in. sieve), adjust the height of the spout to maintain a free fall of the soil of about 1 in. With a steady flow of soil from the spout, move the pouring device in a spiral path from the outside to the center of the mold to form each layer of uniform thickness without segregation.* When the maximum particle size of the sample exceeds 3/8 in., place the soil into the mold by means of the scoop held as closely as possible to the previously placed layer so the soil slides but does not fall from the scoop; restrain the larger particles with the hand where necessary to prevent their rolling from the scoop. Continue filling the mold until the soil rises slightly above the top of the mold, with care that no large particles which project above the top of the mold are placed in the final layer. Using the straightedge, carefully trim the soil surface level with the top of the mold.

(4) If the maximum density of the oven-dried sample is not to be determined,‡ weigh the mold and soil to the nearest 0.01 lb and record the weight on the data sheet; alternatively, the contents of the mold may be emptied into a mixing pan and weighed.

(5) Steps (3) and (4) should be repeated until consistent results (within 1 percent) are attained.

b. Maximum Density Determination With Oven-Dried Sample. The procedure for determining the maximum density shall consist of the following steps:

(1) Proceed in accordance with paragraphs 5a(1) through 5a(3).

(2) Attach the mold to the deck of the vibratory table, if this

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* Static electricity in dry sand can cause bulking similar to that produced by a trace of moisture on the particles; a static-eliminating balance brush can be applied to the equipment in contact with the sand when this effect becomes bothersome.

‡ If the maximum density of the oven-dried sample is to be determined also, proceed in accordance with paragraphs 5b(2) through 5b(7).

had not been done prior to filling the mold with soil (see paragraph 5a(3)).

(3) Place the guide sleeve on the top of the mold and clamp it firmly to the mold.* Lower the surcharge baseplate onto the surface of the soil and remove the handle. Using the hoist if necessary, lower the surcharge weight onto the surcharge baseplate.

(4) It has been determined that for a particular vibrating table, mold, and surcharge assembly, the maximum dry density of a specimen may be obtained at a displacement amplitude (rheostat setting) less than the maximum amplitude of which the apparatus is capable; i.e., dry density may increase with increase in rheostat setting to a setting, beyond which the dry density decreases. Therefore each laboratory should determine for its apparatus the rheostat setting at which maximum density is produced and use this setting for subsequent maximum density testing.‡ A clean, durable, subrounded to rounded material should be used in making these determinations (in both 0.1- and 0.5-cu-ft molds). The test should be performed as given in this appendix, except that the rheostat setting should be increased from zero to 100 in increments of 10 and measurements taken after each period of vibration; the material should be vibrated for a period of 8 min at each rheostat setting and not removed until after the last determination has been made (100 percent rheostat setting). The particle size distribution of the specimen should be determined before and after the test to assess the extent of degradation, if any.

(5) Remove the surcharge weight and guide sleeve from the mold, and obtain dial indicator readings on opposite sides of the surcharge

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* The inside surface of the guide sleeve must align with the inside surface of the mold, so two of the three clamping bolts should be provided with lock nuts, as noted in Figure 2. By properly adjusting and locking these two bolts, the guide sleeve will be drawn automatically into correct alignment when the third bolt is tightened.

‡ It may be desirable to redetermine the optimum rheostat setting at the inception of testing for each major project.

baseplate, as shown in Figure 4b. Record the dial readings on the data sheet.

(6) Remove the surcharge baseplate from the mold and detach the mold from the vibratory table.

(7) Weigh the mold and soil to the nearest 0.01 lb and record the weight on the data sheet; alternatively, the contents of the mold may be emptied into a mixing pan and weighed.

c. (Rescinded)

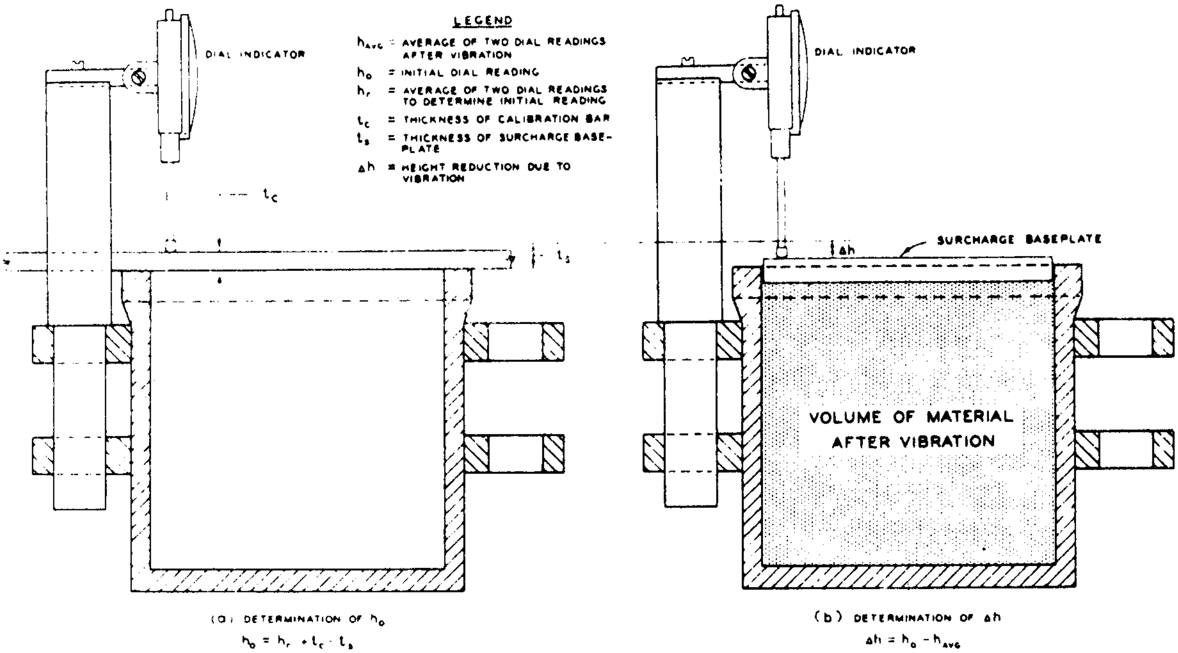

Figure 4. Determination of reduction in sample height due to vibration

6. COMPUTATIONS. The computations shall consist of the following steps:

a. Compute the weight of the dry soil, W_s, by subtracting the weight of the mold, W_m, (or mixing pan) from the weight of the mold (or mixing pan) and soil, W.

b. For the minimum density determination, the volume of the soil, V, is equal to the total inside volume of the mold, V_m.

c. For the maximum density determination, the volume of the soil, V, is equal to the total inside volume of the mold, V_m, minus the volume change, ΔV, caused by vibrating the soil. The volume change shall be computed in the following manner:

(1) Compute the average dial reading, h_{avg}, of the two readings taken after vibrating the soil

(2) Subtract the initial dial reading, h_o, from the average dial reading, h_{avg}, to obtain the height change, Δh.

(3) Compute the volume change by the equation

$$\Delta V = \frac{\Delta h}{12} \times A_m$$

where

ΔV = volume change, cu ft

Δh = height change, in.

A_m = end area of mold, sq ft

d. Compute the minimum and maximum densities to the nearest 0.1 lb per cu ft by the equation

$$\gamma_d = \frac{W_s}{V}$$

where

γ_d = dry density, lb per cu ft
W_s = weight of dry soil, lb
V = volume of soil, cu ft

e. If the in-place density of the soil is known, the relative density, D_d, can be computed by the equation given in paragraph 1, or determined graphically by means of Figure 5.

7. POSSIBLE ERRORS. Following are possible errors that would cause inaccurate determinations of relative density:

a. General. (1) Test not appropriate to type of soil. The relative density is meaningful only for cohensionless materials; if a soil has any appreciable dry strength, the methods for determining the minimum and maximum densities described in this Appendix are not applicable.

(2) Material segregated while being processed.

(3) Gain in moisture of oven-dried material before or during testing. A small amount of moisture in the soil can cause erroneous measurements of the minimum density and, to a much lesser degree, of the maximum density.

(4) Molds not accurately calibrated.

b. Minimum Density Determination. (1) Disturbance of mold during filling. Inadvertent jarring of the mold or impact of the falling particles will increase the measured minimum density.

(2) Segregation of material while filling mold.

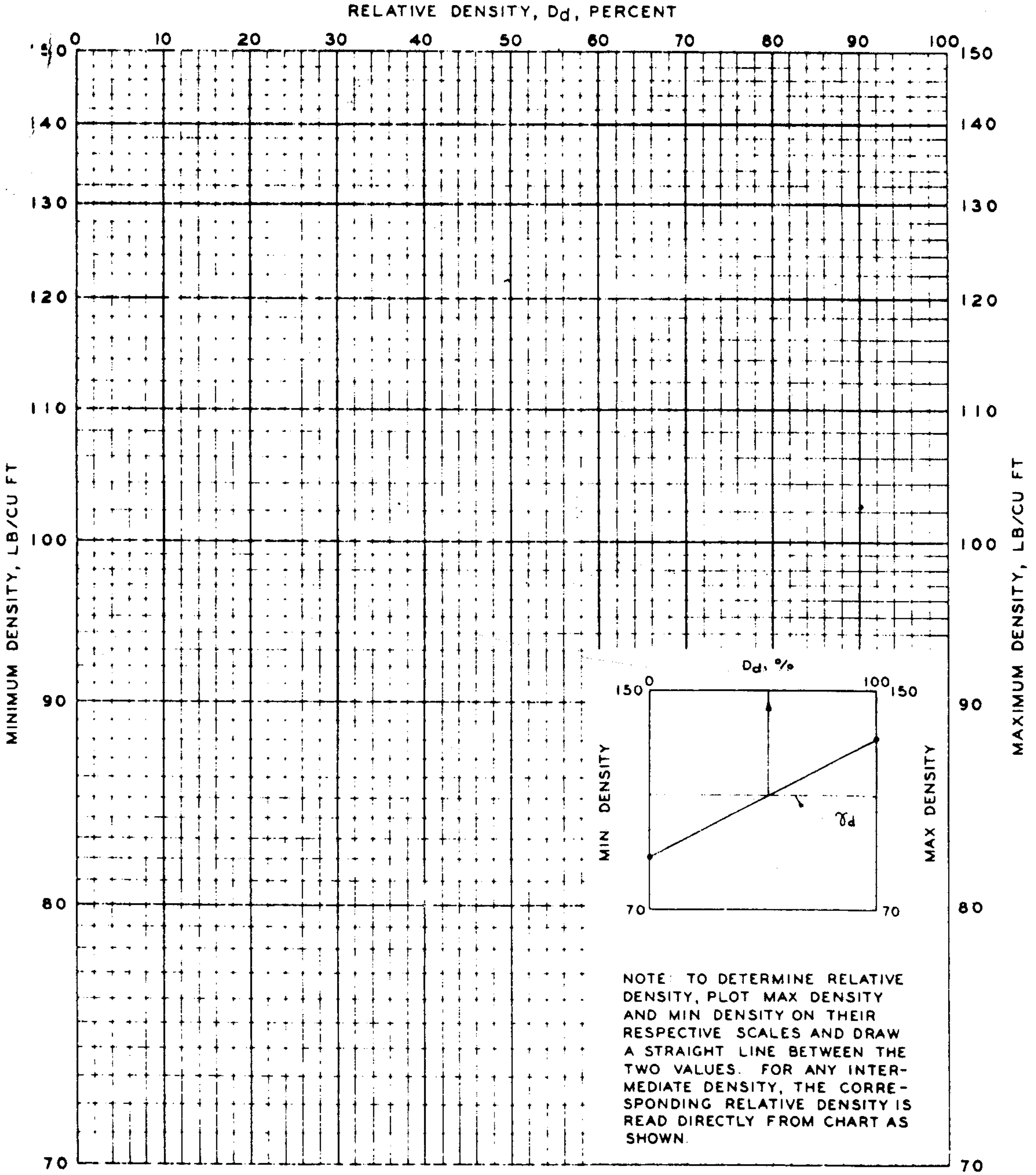

Figure 5. Graphical determination of relative density

(3) Loss of material from mold before weighing. To prevent spilling any material before the mold and contents can be weighed, rap the side of the mold carefully to settle the contents.

c. Maximum Density Determination. (1) Insufficient amplitude of vibratory table under load. Measurements should be made at least once to verify that the requirements given in paragraph 2f are satisfied.

(2) (Rescinded)

(3) Loss of fine material from mold during vibration. Fine particles may escape from the mold during vibration as dust.

(4) Misalignment of guide sleeve with mold. The adjustment of the clamping bolts must be checked periodically to ascertain that misalignment of the guide sleeve will not cause binding of the surcharge weight.

RELATIVE DENSITY

MINIMUM AND MAXIMUM DENSITY DETERMINATIONS

DATE ______

PROJECT ______

BORING NO. ______ SAMPLE NO. ______

CLASSIFICATION						
MOLD NO.	MOLD DIAMETER, IN. =		WEIGHT OF MOLD, W_m, LB =			
VOLUME OF MOLD, V_m, CU FT =			END AREA OF MOLD, A_m, SQ FT =			
MINIMUM DENSITY						
TRIAL NO.			1	2	3	4
WEIGHT LB	MOLD (OR TARE) AND SOIL, DRY	W				
	MOLD (OR TARE)	W_m				
	SOIL, DRY	W_s				
MIN DRY DENSITY, LB/CU FT = W_s/V		γ_d				
MIN DRY DENSITY, AVERAGE			LB/CU FT			
MAXIMUM DENSITY						
METHOD USED						
TRIAL NO:			1	2	3	4
HEIGHT, IN.	LEFT DIAL READING	h_L				
	RIGHT DIAL READING	h_R				
	AVERAGE DIAL = $(h_L + h_R)/2$	h_{avg}				
	INITIAL DIAL READING	h_o				
	HEIGHT CHANGE = $h_o - h_{avg}$	Δh				
VOL CU FT	VOLUME CHANGE = $(\Delta h/12) \times A_m$	ΔV				
	VOLUME OF SOIL = $V_m - \Delta V$	V				
WEIGHT LB	MOLD (OR TARE) AND SOIL, DRY	W				
	MOLD (OR TARE)	W_m				
	SOIL, DRY	W_s				
MAX DRY DENSITY, LB/CU FT = W_s/V		γ_d				
MAX DRY DENSITY, AVERAGE			LB/CU FT			

REMARKS ______

TECHNICIAN ______ COMPUTED BY ______ CHECKED BY ______

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PLATE XII-1